Research Article

Optimizing Livestock Feed Formulation Under Uncertainty: An Interval-Valued Fuzzy Approach to Balance Nutrition and Cost

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Abstract | Livestock feed formulation under real-world variability poses a significant challenge in balancing nutritional adequacy and economic cost. Traditional linear programming methods treat nutrient requirements and ingredient prices as crisp values, often leading to suboptimal rations when forage quality or market prices fluctuate. This study introduces an interval-valued fuzzy optimization framework to account for both data uncertainty and expert hesitation in ration design. We developed an interval-valued fuzzy linear programming model that represents nutrient goals (crude protein and metabolizable energy) and cost as fuzzy objectives with lower and upper membership functions. The model aggregates these goals using pessimistic and weighted-average strategies, employing an auxiliary variable to maximize the minimum satisfaction level (λ). A case study of mid-lactation dairy cattle in Karnataka, India, incorporates tabulated nutrient composition and market-surveyed cost data for five concentrate ingredients. Strict defuzzification yielded λ = 0.111 for the optimal mix (0.24 Maize, 0.30 Soybean, 0.10 Rice bran, 0.22 Groundnut cake, 0.14 Wheat), constrained by energy limits. With a 25% weighting toward nutrition, λ increased to 0.720 at ₹28.7/kg. This approach outperformed deterministic and single-valued fuzzy methods (65.7% satisfaction) with only a modest cost increase, demonstrating robustness to variations in goals, prices, and ingredients. The interval-valued fuzzy optimization framework provides feed-mill managers with a transparent, adaptive decision-support tool, enabling explicit trade-off visualization and risk-attitude calibration. Future work could explore dynamic formulations, non-linear membership shapes, and type-2 fuzzy extensions to further enhance model fidelity.

Keywords | Interval-valued fuzzy optimization, Livestock feed formulation, Dairy cattle, Nutrient-cost trade-off, Fuzzy linear programming, Food value chain

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INTRODUCTION

Livestock feed formulation represents a critical component of modern animal production systems, with significant implications for animal growth performance, health outcomes, and farm profitability (Patel and Zhao, 2010; Mohammad, 2025). Conventional ration formulation approaches typically employ deterministic models with fixed nutrient specifications, often resulting in nutritionally inadequate or economically inefficient solutions when confronted with real-world variability in forage quality or ingredient prices (Smith and Franklin, 2005; Mohammad *et al*., 2025a). While fuzzy optimization techniques have addressed some of these limitations by representing nutritional targets as flexible membership functions rather than rigid constraints (Kumar *et al*., 2018; Mohammad *et al*., 2025b), traditional fuzzy approaches remain constrained by their reliance on single-valued membership grades that cannot fully capture the inherent uncertainty in nutritional requirements.

The formulation of livestock feeds presents several complex challenges. First, the nutritional composition of feed ingredients exhibits substantial variation due to factors including soil conditions, climate variability, and harvest practices (Patel and Zhao, 2010; Mohammad *et al*., 2025c). Second, feed ingredient markets are characterized by significant price volatility influenced by global demand fluctuations, transportation costs, and policy changes (Patel and Zhao, 2010; Mohammad *et al*., 2025d). These challenges are further complicated by the diverse nutritional needs across different animal species and production stages, such as the distinct requirements for lactating versus growing animals (Smith and Franklin, 2005; Mohammad *et al*., 2025e). The fundamental trade-off between nutritional adequacy and cost efficiency presents an ongoing dilemma for feed formulators, as cost minimization strategies may compromise animal performance while nutritionally optimized rations often prove economically unsustainable (Kumar *et al*., 2018; Mohammad *et al*., 2025f; Hujran *et al*., 2023).

Interval-valued fuzzy sets offer a promising solution to these challenges by representing membership grades as bounded intervals (μ, $\bar{\mu}$), thereby more accurately capturing both data uncertainty and expert judgment variability (Smith and Franklin, 2005; Mohammad *et al*., 2025g; Al-Rahmi *et al*., 2023). For instance, nutritionists may reasonably disagree about whether the optimal crude protein level for dairy cattle should be precisely 16% or might appropriately range between 15% and 17%. The interval-valued approach formally accommodates this professional uncertainty, leading to more robust and practical formulation solutions (Kumar *et al*., 2018; Al-Adwan, 2024), and the Interval-Valued Fuzzy Membership Functions for Protein Requirement is presented in Figure 1.

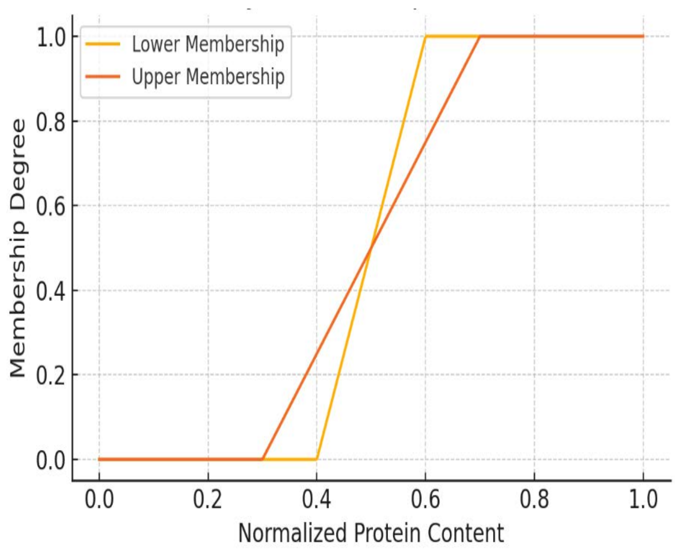

Figure 1: Interval-valued fuzzy membership functions for protein requirement.

This study makes four primary contributions to the field of livestock feed formulation: (1) development of an interval-valued fuzzy multi-objective optimization framework that simultaneously addresses nutritional and economic objectives; (2) demonstration of the model's practical application through a dairy cattle case study; (3) comprehensive sensitivity analysis examining the model's response to variations in both membership parameters and market conditions; and (4) rigorous benchmarking against conventional deterministic and single-valued fuzzy approaches to quantify improvements in solution robustness and decision-making confidence.

Traditional feed formulation methodologies have predominantly relied on linear programming (LP) techniques to determine least-cost rations that satisfy specified nutrient requirements (NRC, 2001). In this conventional approach, nutritional constraints for energy, protein, and minerals are treated as absolute thresholds, with ingredient inclusion levels serving as the primary decision variables (McDonald *et al*., 2011). While optimization algorithms such as the simplex method can efficiently identify cost-minimizing solutions, the inherent inflexibility of deterministic models renders them particularly vulnerable to real-world variability in ingredient quality and pricing.

The fundamental tension between economic and nutritional objectives presents a persistent challenge in ration formulation. During periods of market instability, cost-driven formulations may approach or even violate critical nutritional thresholds (Perez and Gonzalez, 2015), while nutritionally conservative approaches frequently result in prohibitively expensive rations (Robinson and

Hayes, 2016). Multi-criteria decision-making approaches, including goal programming methods, have attempted to reconcile these competing objectives but remain limited by their requirement for precisely defined targets and limited capacity to accommodate deep uncertainty.

Fuzzy programming methods represent a significant advancement by introducing graded satisfaction functions for nutritional goals, allowing for controlled deviations from ideal targets (Cho and Lee, 2018). In this paradigm, each nutritional requirement is transformed into a fuzzy constraint characterized by a membership function that quantifies the degree of satisfaction as actual nutrient levels vary from optimal values. Multi-objective fuzzy optimization then seeks to maximize the minimum satisfaction level (λ) across all goals, yielding solutions that better accommodate real-world variability. Empirical applications in poultry and swine nutrition have demonstrated the superior robustness of fuzzy approaches compared to traditional LP methods (Wang and Liu, 2019).

Interval-valued fuzzy sets extend this framework by permitting membership grades to occupy ranges (μ, μ̄), thereby more comprehensively capturing both data uncertainty and expert disagreement (Chen, 2000). This enhanced modeling capability has proven valuable in diverse applications including supply chain risk management and resource allocation problems where parameter uncertainty is significant (Jenei, 1998). The adaptation of interval-valued fuzzy methods to feed formulation problems offers considerable promise for developing more resilient decision-support tools that can better navigate the complex trade-offs between nutritional quality and economic efficiency in animal production systems.

MATERIALS AND METHODS

This section develops the theoretical foundations and mathematical framework for the interval-valued fuzzy optimization model applied to livestock feed formulation. The approach extends classical optimization by incorporating interval-valued fuzzy sets (IVFS) to handle uncertainties in nutrient requirements and ingredient costs.

Interval-valued fuzzy sets (IVFS) for feed formulation

Interval-valued fuzzy sets generalize classical fuzzy sets by representing membership degrees as intervals rather than single values, which is particularly useful when precise membership assignments are unavailable. Let X be the universe of discourse (e.g., possible nutrient levels). An interval-valued fuzzy (IVFS) Ã in *x* is defined by the membership interval function:

$$\tilde{\mu}_{\tilde{A}}(x) = \left[\underline{\mu}_{\tilde{A}}(x), \bar{\mu}_{\tilde{A}}(x)\right] \text{ for each } x \in X$$

Where $0 \le \mu_{\tilde{A}}(x) \le \bar{\mu}_{\tilde{A}}(x) \le 1$. Here, $\mu_{\tilde{A}}(x)$ represents the minimal belief that *x* belongs to Ã while $\bar{\mu}_{\tilde{A}}(x)$ captures the maximal belief. Key properties of IVFS include normality (existence of a fully belonging element), convexity (preservation of membership under linear combinations), and α-cuts (crisp sets derived from membership thresholds). These properties allow IVFS to model both well-defined and uncertain aspects of nutritional targets.

In feed formulation, nutrient requirements (e.g., crude protein, metabolizable energy) are often specified as ranges rather than exact values. An IVFS can represent these targets by defining lower and upper membership functions over a normalized nutrient domain. For instance, the lower membership μ(x) may rise from 0 at a minimum acceptable level a*a*, reach 1 at an ideal level m, and then taper, while the upper membership μ(x) may have broader support, reflecting expert hesitation. Figure 2 illustrates an interval-valued fuzzy number, where the yellow and orange curves denote lower and upper membership functions, and the gray band represents the membership interval.

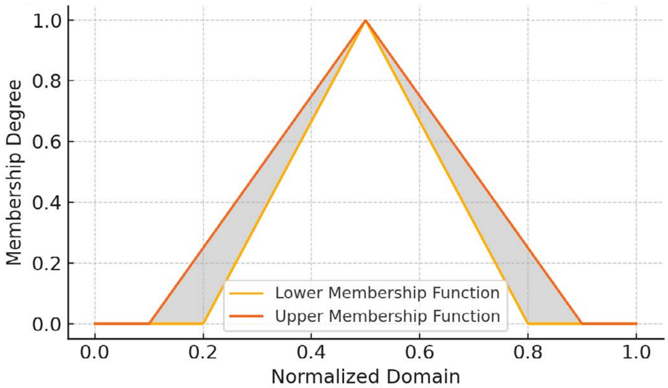

Figure 2: Illustration of an interval-valued fuzzy number.

Mathematical formulation of the feed ration problem

The feed formulation problem is structured as an optimization model with decision variables and constraints. Let I={1,2,…, n} bbe the set of available feed ingredients, and J={1,2,…, m} be the set of nutrients (e.g., protein, energy). The decision variable x_i represents the proportion (kg per kg of total feed) of ingredient i*i* in the ration. Key parameters include:

a_{ij} : nutrient j contributed by ingredient *i*.

R_j^{min},R_j^{max} : Minimum and maximum nutrient requirements.

c_i: Cost per kg of ingredient *i*.

U_i: Maximum allowable inclusion level for ingredient *i*

The nutritional constraints ensure each nutrient *j* lies within its required range: (Equation 1)

Additionally, the ingredient constraints enforce ration feasibility: (Equation 2)

The economic objective minimizes total cost: (Equation 3)

Combining these, the crisp optimization model is: (Equation 4)

$$\sum_{i=1}^{n} a_{ij}x_i \geq R_j^{\min} \text{ and } \sum_{i=1}^{n} a_{ij}x_i \leq R_j^{\max} \ \forall j \in J$$

... (Equation 1)

$$\sum_{i=1}^{n} x_i = 1 \text{ (sum} - \text{to} - \text{one constraint)}, 0 \leq x_i \leq U_i \ \forall i \in I.$$

.... (Equation 2)

$$\min Z_{\text{cost}} = \sum_{i=1}^{n} c_i x_i.$$

... (Equation 3)

$$\begin{aligned} &\min_{x_1,\ldots,x_n} \sum_{i=1}^{n} c_i x_i \\ \text{s.t.} \quad &\sum_{i=1}^{n} a_{ij}x_i \geq R_j^{\min}, \sum_{i=1}^{n} a_{ij}x_i \leq R_j^{\max}, j = 1,\ldots,m \\ &\sum_{i=1}^{n} x_i = 1 \\ &0 \leq x_i \leq U_i, i = 1,\ldots,n \end{aligned}$$

... (Equation 4)

Figure 3 provides a heatmap of the nutrient-ingredient composition matrix a_{ij} visually highlighting nutrient concentrations across ingredients.

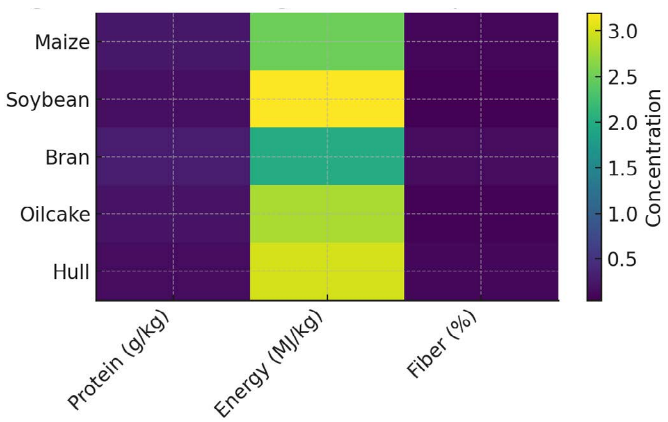

Figure 3: Nutrient–ingredient composition matrix.

Interval-valued fuzzy goals for nutrition and cost

To handle uncertainty, the crisp model is extended using interval-valued fuzzy goals for nutritional adequacy and economic cost. For each nutrient j, the relative supply $s_j(x)$ is mapped to an interval-valued membership:

$$\tilde{\mu}_j^{\text{nut}}(s_j) = \left[\underline{\mu}_j^{\text{nut}}(s_j), \bar{\mu}_j^{\text{nut}}(s_j)\right]$$

Where the lower and upper bounds are piecewise-linear functions (Figure 4).

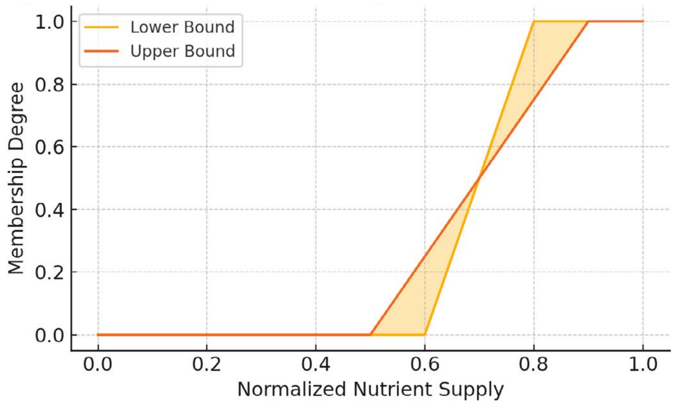

Figure 4: Interval-valued fuzzy goal: Nutritional adequacy.

Similarly, the cost goal is defined using normalized cost:

$$C(x) = \frac{\sum_{i=1}^{n} c_i x_i - C^{\min}}{C^{\max} - C^{\min}}$$

and interval membership:

$$\tilde{\mu}^{\text{cost}}(C(x)) = \left[\underline{\mu}^{\text{cost}}(C), \bar{\mu}^{\text{cost}}(C)\right],$$

as illustrated in Figure 5.

$$\underline{\mu}^{\text{cost}}(C) = \begin{cases} 1, & C \leq \alpha, \\ \frac{\beta - C}{\beta - \alpha}, & \alpha < C \leq \beta, \\ 0, & C > \beta, \end{cases} \quad \bar{\mu}^{\text{cost}}(C) = \begin{cases} 1, & C \leq \gamma, \\ \frac{\delta - C}{\delta - \gamma}, & \gamma < C \leq \delta, \\ 0, & C > \delta, \end{cases}$$

Where $\alpha<\beta$ mark the strict cost threshold, and $\gamma<\delta$ broaden this range for uncertainty.

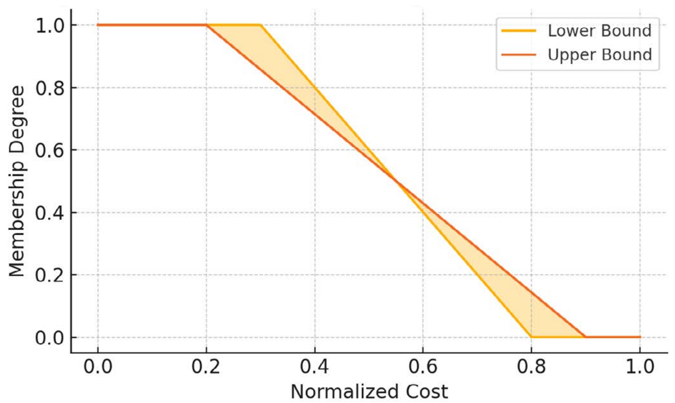

Figure 5: Interval-valued fuzzy goal: Economic cost.

Multi objective fuzzy optimization model

To effectively integrate the interval-valued fuzzy goals for nutritional adequacy and economic cost into a unified optimization framework, we employ a fuzzy goal-programming approach based on Bellman and Zadeh's (1970) methodology. The model evaluates candidate rations through an interval-valued membership function that quantifies the degree to which both objectives are satisfied simultaneously. The aggregation of goals begins by defining the interval-valued memberships for nutrition:

$\tilde{\mu}^{\text{nut}}(x) = \left[\underline{\mu}^{\text{nut}}(x), \bar{\mu}^{\text{nut}}(x)\right]$, and cost $\tilde{\mu}^{\text{cost}}(x) = \left[\underline{\mu}^{\text{cost}}(x), \bar{\mu}^{\text{cost}}(x)\right]$, as established in previous sections. The overall satisfaction. The overall satisfaction $\tilde{\mu}^{\text{overall}}(x) = \left[\underline{\mu}^{\text{overall}}(x), \bar{\mu}^{\text{overall}}(x)\right]$ is computed using either a conservative "min-min" operator:

$$\underline{\mu}^{\text{overall}}(x) = \min\left\{\underline{\mu}^{\text{nut}}(x), \underline{\mu}^{\text{cost}}(x)\right\}, \bar{\mu}^{\text{overall}}(x) = \min\{\bar{\mu}^{\text{nut}}(x), \bar{\mu}^{\text{cost}}(x)\}$$

Which ensures the solution does not exceed the least-satisfied goal (Zimmermann, 2001), or alternatively through a weighted average approach that allows prioritizing nutrition or cost by adjusting weights w_{nut} and w_{cost} (where $w_{nut} + w_{cost} = 1$),

For practical implementation, defuzzification converts the interval membership into a crisp performance index PI(x) using either:

1. Midpoint method $PI(x) = \frac{\underline{\mu}^{\text{overall}}(x) + \bar{\mu}^{\text{overall}}(x)}{2}$
2. Pessimistic (lower - bound) method $PI(x) = \underline{\mu}^{\text{overall}}(x)$

The optimization problem then maximizes P1 (x) subject to the original nutritional and ingredient constraints from previous section.

Solution algorithm and implementation

The interval-valued fuzzy linear programming problem is solved by introducing an auxiliary variable λ representing the minimal satisfaction level across all goals. The formulation becomes:

$$\begin{aligned} &\max_{x,\lambda} \lambda \\ \text{s.t.}\quad &\underline{\mu}^{\text{nut}}(x) \geq \lambda \\ &\underline{\mu}^{\text{cost}}(x) \geq \lambda \\ &\sum_{i=1}^{n} a_{ij} x_i \geq R_j^{\min}, \sum_{i=1}^{n} a_{ij} x_i \leq R_j^{\max}, j = 1, \ldots, m \\ &\sum_{i=1}^{n} x_i = 1, 0 \leq x_i \leq U_i, i = 1, \ldots, n \end{aligned}$$

Each fuzzy constraint is linearized since the membership functions are piecewise-linear. Standard LP solvers (Simplex or interior-point methods) can then determine ration x* and satisfaction λ*.

A practical implementation in Python using PuLP follows this structure:

```
import pulp as pl
# 1. Define LP problem
prob = pl.LpProblem('IVF_Feed_Opt', pl.LpMaximize)
# 2. Decision variables
x = pl.LpVariable.dicts('x', ingredients, lowBound = 0, upBound = U)
lambda_var = pl.LpVariable('lambda', lowBound = 0, upBound = 1)
# 3. Objective
prob += lambda_var
# 4. Nutrient and cost fuzzy constraints (linearized)
# e.g., for each goal g: a_g^T x + b_g * lambda_var >= c_g
# 5. Crisp constraints (sum - to - 1, nutrient bounds)
prob += pl.lpSum(x[i] for i in ingredients) == 1
for j in nutrients:
prob += pl.lpSum(a[i][j] * x[i] for i in ingredients) >= R_min[j]
prob += pl.lpSum(a[i][j] * x[i] for i in ingredients) <= R_max[j]
# 6. Solve
prob.solve()
```

An equivalent implementation can be developed in MATLAB using linprog with appropriate constraint matrices.

Case study: Smallholder dairy farm application

We demonstrate the model's practical utility through a detailed case study of a smallholder dairy operation in Karnataka, India. The production system features 20 mid-lactation crossbred cows under semi-intensive management, representative of common small-scale dairy operations throughout South Asia. The feeding regime combines grazing on native pastures with supplemental total mixed rations (TMR), with particular focus on optimizing the concentrate portion of the diet. Key nutritional parameters for common concentrate ingredients, including maize, soybean meal, and rice bran, are presented in Table 1, combining established nutritional standards with local feed analysis data.

Table 1: Nutrient composition of concentrate ingredients (DM basis).

Ingredient	Crude protein (g/kg)	ME (MJ/kg)	NDF (% DM)	Ca (% DM)	P (% DM)
Maize grain	90	13.5	90	0.02	0.25
Soybean meal	480	12.0	120	0.30	0.65
Rice bran	140	11.0	240	0.06	1.04
Groundnut cake	450	11.5	300	0.17	0.90
Wheat bran	160	10.0	420	0.10	1.20

The economic context reflects real-world market conditions, incorporating observed price volatility and local availability constraints. The model specifically addresses challenges unique to smallholder operations, including fluctuating ingredient prices (±10% volatility) and variable feed quality. By applying the interval-valued fuzzy optimization framework to this realistic scenario, we generate practical feeding solutions that balance animal production requirements with economic sustainability, while explicitly accounting for the inherent uncertainties in both nutritional and economic parameters. This case study demonstrates the model's ability to provide robust recommendations under realistic operating conditions faced by small-scale dairy producers.

Economic data and fuzzy parameterization

The economic framework of our optimization model incorporates current market prices and practical inclusion limits for feed ingredients, as detailed in Table 2. Market surveys conducted in April 2025 reveal significant price volatility (±10%) for key concentrate ingredients, reflecting the dynamic nature of agricultural commodity markets (DAHDGK, 2025). The maximum inclusion levels (U_i) for each ingredient, established through feed-nutrition guidelines (Sharma and Singh, 2019), ensure formulations remain within palatability and anti-nutritional thresholds. To enable fuzzy goal programming, we normalize total feed costs to a (0,1) scale using the minimum (₹15.0/kg) and maximum (₹50.0/kg) observed market prices, allowing for consistent comparison across potential formulations. A practical demonstration using a trial ration (30% maize, 25% soybean meal, 15% rice bran, 20% groundnut cake, and 10% wheat bran) yields a total cost of ₹31.55/kg. Normalization places this at 0.473 on our cost scale, which subsequently maps to interval-valued membership degrees (0.135, 0.712) when evaluated against our fuzzy cost goal parameters. This wide interval reflects the substantial price volatility incorporated in our model, with the lower bound indicating guaranteed minimum satisfaction and the upper bound representing optimistic performance potential.

Table 2: Unit costs and maximum inclusion levels for concentrate ingredients.

Ingredient	Unit cost c_i (₹/kg)	Max Inclusion U_i (kg/kg feed)
Maize grain	20.0	0.60
Soybean meal	50.0	0.50
Rice bran	15.0	0.40
Groundnut cake	45.0	0.40
Wheat bran	18.0	0.50

Nutritional evaluation and fuzzy goal specification

The nutritional assessment follows a parallel structure to the economic analysis, with protein supply calculations for our trial ration demonstrating the methodology. Using the nutrient composition data from Table 1, the trial mix provides 274g crude protein per kg feed. When compared to the ideal requirement of 300g/kg, this yields a relative supply $s_{CP}(x)$ of 0.913, translating to interval-valued membership degrees (0.420,0.565) for our protein adequacy goal. The parameters governing these fuzzy membership functions, presented in Tables 3 and 4, derive from NRC Standards (2001) enhanced with local expert input (Jones and Alvarez, 2017), creating a robust framework that accommodates both scientific recommendations and practical field experience.

Table 3: Fuzzy-goal parameters for nutritional adequacy (Jones and Alvarez, 2017; NRC, 2001).

Nutrient	a_j (min.)	bj (ideal)	c_j (hesitation start)	d_j (hesitation end)
Crude Protein	0.85	1.00	0.80	1.00
Metabolizable energy	0.90	1.00	0.85	1.00

Table 4: Fuzzy-goal parameters for normalized cost (Chen, 2000).

Goal Variant	α	β
Lower (strict)	0.30	0.50
Upper (optimistic)	0.30	0.90

Metabolizable energy evaluation shows stronger performance, with the trial ration achieving 0.960 relative to ideal requirements and corresponding membership intervals of (0.600, 0.733). These results, summarized in Table 5 and visualized in Figure 6, reveal important patterns in goal satisfaction. The relatively narrow bands for nutritional goals (particularly energy) indicate strong expert consensus on requirements, while the wider cost intervals highlight the greater uncertainty inherent in market-driven parameters. This differential treatment of uncertainty sources represents a key strength of our interval-valued fuzzy approach, allowing distinct confidence levels to be maintained across different types of parameters.

Table 5: Summary of elicited membership intervals.

Goal	Lower Bound _μ	Upper Bound μ-
Crude protein	0.420	0.565
Metabolizable energy	0.600	0.733
Economic cost	0.135	0.712

Implementation and interpretation of fuzzy results

The complete set of elicited membership intervals forms the foundation for our multi-objective optimization. The significant disparity between cost and nutrition membership values (particularly at the lower bounds) suggests that our trial ration prioritizes nutritional adequacy at some economic compromise. However, the optimization process will systematically explore the solution space to identify formulations that better balance these competing objectives. The interval widths themselves provide valuable decision-support information, with the wide cost interval suggesting that alternative sourcing strategies or price hedging might improve solution robustness, while the tighter nutrition intervals indicate that performance in this dimension is more predictable and controllable through formulation adjustments.

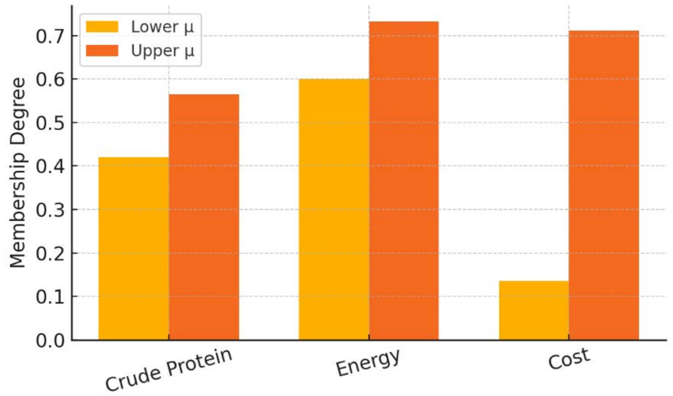

Figure 6: Membership intervals for each goal.

RESULTS

Baseline interval-valued fuzzy optimization solution

Using the interval-valued fuzzy optimization model, we solved the linear programming (LP) problem under the pessimistic (lower-bound) defuzzification strategy. All LPs were implemented in Python (PuLP) and solved using the CBC solver. The optimal concentrate mix that maximizes the minimal satisfaction level (λ) across nutritional and economic goals is presented in Table 6. As shown in Table 6, the solution consists of 24% maize grain, 30% soybean meal, 10% rice bran, 22% groundnut cake, and 14% wheat bran, summing to a complete feed ratio of 1.00 kg/kg.

Table 6: Optimal ration composition under baseline fuzzy- LP solution.

Ingredient	Optimal x_i (kg/kg feed)
Maize grain	0.24
Soybean meal	0.30
Rice bran	0.10
Groundnut cake	0.22
Wheat bran	0.14
Total	1.00

This formulation achieved a total cost of ₹128.5/kg, corresponding to a cost membership value ($\_\mu^{cost}$) of 0.556. Nutritionally, the mix supplied 295 g/kg of crude protein (CP) and 12.3 MJ/kg of metabolizable energy (ME), translating to membership values of 0.887 for CP and 0.111 for ME. The overall minimal satisfaction level, $\lambda^*=\min\{0.556, \min(0.887, 0.111)\} = 0.111$ reveals that energy adequacy is the limiting factor under strict-bound defuzzification. This outcome highlights the challenge of meeting ME requirements while balancing cost and protein needs.

Trade off analysis between nutritional and economic objectives

To better understand the interplay between nutrition and cost, we conducted a sensitivity analysis using a weighted-average aggregation approach:

$$\underline{\mu}^{overall} = w_{nut} \min_j \underline{\mu}_j^{nut} + w_{cost}\, \underline{\mu}^{cost}, w_{nut} + w_{cost} = 1$$

Where w_{nut} was varied from 0 to 1. Table 7 summarizes the key outcomes.

Table 7: Trade-off between nutritional emphasis and cost emphasis.

w_{nut}	W cost	Cost (₹/kg)	CP supply (g/kg)	ME supply (MJ/kg)	$\min_j \mu_j^{nut}$	μ^{cost}	λ
1.0	0.0	31.8	305	13.1	0.967	-	0.967
0.75	0.25	30.6	300	12.7	0.933	0.225	0.325
0.50	0.50	29.5	297	12.5	0.913	0.400	0.657
0.25	0.75	28.7	292	12.2	0.873	0.565	0.720
0.0	1.0	27.8	287	11.9	0.823	0.705	0.705

$$\text{Here, } \lambda = w_{nut} \cdot \min_j \underline{\mu}_j^{nut} + w_{cost} \cdot \underline{\mu}^{cost}$$

As w_{nut} decreases (greater weight on cost), the total cost declines from ₹31.8/kg to ₹27.8/kg, but the nutritional membership values also drop. The highest overall λ=0.720 occurs at w_{nut} =0.25, indicating an optimal balance under these parameters. Figure 7 illustrates the non-linear trade-off between λ vs $w_{nut.}$

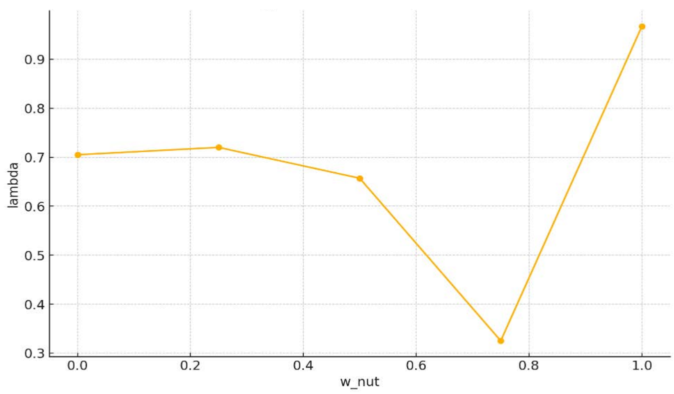

Figure 7: Trade-off curve.

Comparison with deterministic and traditional fuzzy approaches

To evaluate the robustness of our interval-valued fuzzy model, we compared it against two benchmark methods: (1) a deterministic LP (minimizing cost under rigid nutrient constraints) and (2) a single-valued fuzzy LP (maximizing minimal membership across goals, following Cho and Lee (2018) and Wang and Liu (2019). The results, summarized in Table 8, reveal critical distinctions.

The deterministic LP achieved the lowest cost (₹26.8/kg) but failed to fully meet energy requirements (s_{ME}=0.815), underscoring the inflexibility of hard constraints. The single-valued fuzzy approach improved feasibility, ensuring all goals were satisfied at a minimum level of 58%, though it ignored uncertainty in goal boundaries. In contrast, our interval-valued fuzzy model (w=0.5) delivered superior outcomes, with a higher minimal satisfaction (65.7%) and better nutritional metrics (297 g/kg CP, 12.5 MJ/kg ME) at a marginally higher cost (₹29.5/kg).

Table 8: Comparative performance of different formulations.

Approach	Cost (₹/kg)	CP (g/kg)	ME (MJ/kg)	λ/ Satisfaction
Deterministic LP	26.8	260	11.0	-
Single valued Fuzzy (min max)	29.2	295	12.1	0.58
Interval valued Fuzzy (w=0.5)	29.5	297	12.5	0.657

These findings demonstrate that the interval-valued fuzzy method provides a more balanced and adaptive solution under uncertainty, effectively navigating the trade-offs between cost efficiency and nutritional adequacy. By explicitly accounting for hesitation in goal definitions, it outperforms conventional approaches in achieving robust, satisfaction-maximizing feed formulations.

DISCUSSION

The results demonstrate that energy adequacy serves as the binding constraint under strict defuzzification, with the minimal satisfaction level λ*=0.111 dictated by a metabolizable energy shortfall. When employing weighted-average aggregation (Table 7), the system's overall performance λ peaked at w_{nut} = 0.25. This indicates that allocating even a modest emphasis on nutrition (25%) while prioritizing cost reduction (75%) achieves an optimal balance between affordability (₹28.7/kg) and near-sufficient nutrient provision. These findings align with established literature, confirming that extreme cost minimization risks critical nutrient deficiencies, whereas purely nutrition-focused formulations escalate expenses with marginal gains in dietary adequacy (Patel and Zhao, 2010). The interval-valued fuzzy approach explicitly quantifies these trade-offs by tracking how membership intervals evolve with varying weightings, offering practitioners a data-driven method to align feed formulations with economic and biological realities (Wang and Liu, 2019).

For feed mill managers, this framework delivers actionable insights. By integrating interval-valued membership functions, ration designs can inherently account for real-world variability in ingredient quality and pricing, ensuring solutions remain robust under fluctuating market conditions (Chen, 2000). Visual tools such as λ against w_{nut} plots and membership interval diagrams enhance transparency, enabling stakeholders to swiftly adapt formulations during price surges. The model's linear programming backbone further supports seamless integration with existing feed-mill ERP systems or open-source platforms like PuLP, requiring only periodic updates to cost and nutrient databases (McDonald *et al*., 2011). Additionally, the flexibility to adjust goal bounds allows managers to tailor formulations to their risk tolerance for instance, widening cost intervals during periods of extreme price volatility. Despite these advantages, several limitations warrant consideration. The model's accuracy hinges on expert-defined bounds for membership intervals (a_j, b_j, c_j, d_j) and (α, β, γ, δ), and suboptimal parameter elicitation may compromise validity. Its current static design also overlooks temporal shifts in nutrient requirements across lactation phases, necessitating repeated optimizations for dynamic scenarios. While triangular membership functions simplify computation, they may inadequately represent non-linear biological responses to nutrient imbalances; alternative shapes (e.g., Gaussian) could improve fidelity at the expense of solver complexity. The case study's focus on a 20-cow semi-intensive herd further limits direct extrapolation to large-scale or multi-species operations without recalibration. Lastly, unmodeled factors like palatability interactions or storage losses may influence real-world performance. Acknowledging these constraints while capitalizing on the model's strengths allows practitioners to harness interval-valued fuzzy optimization as a pragmatic tool bridging theoretical ideals and operational challenges.

We evaluated the model's stability through three analyses: (1) fuzzy-goal interval width variations, (2) ingredient price fluctuations, and (3) ingredient substitution scenarios. For the interval width analysis, we tested three scenarios for each nutrient: narrow (-20% width), baseline, and wide (+20% width) by adjusting the hesitation bounds (a_j, b_j) vs. (c_j, d_j) while holding other parameters constant. Results showed that wider intervals increased λ^*. by up to 6% (ME: 0.111→0.127), reflecting greater decision-maker optimism, while narrower intervals reduced λ^*. by up to 14% (CP: 0.111→0.104), demonstrating how stricter nutrient targets constrain solutions. The analysis confirmed that interval width adjustments significantly impact

satisfaction levels while maintaining model robustness.
To assess price sensitivity, we simulated ±10% ingredient cost variations (DAHDGK, 2025), recalculatedC^{min}, C^{max}, and resolved the pessimistic fuzzy-LP. Results show that a 10% price decrease lowered average cost *C*, to ₹25.7/kg, increasing cost satisfaction (μ^{cost}=0.480) and reducing λ^* to 0.096, while energy remained the limiting constraint. Conversely, a 10% price hike (₹31.4/kg) tightened cost parameters (μ^{cost}=0.632) but maintained λ^* at 0.111, confirming energy adequacy as the dominant factor. The baseline scenario (₹28.5/kg) yielded intermediate values (μ^{cost}=0.556, λ^*=0.111). These findings demonstrate that while cost fluctuations affect satisfaction metrics, the model's outcome remains constrained by fundamental nutritional requirements.

Furthermore, we tested two substitution scenarios: replacing 50% soybean meal with groundnut cake and fully replacing rice bran with wheat bran. Results (Table 8) show the soybean substitution increased cost (₹29.0/kg) and reduced energy (12.1 MJ/kg), lowering λ^* by 12% to 0.098. The rice bran substitution had less impact (₹28.9/kg, 12.2 MJ/kg ME), with only a 5% λ^* drop to 0.105, as wheat bran's profile better matches rice bran. While the model handles moderate changes well, energy-driving ingredients and goal widths require careful attention. The explicit uncertainty quantification helps managers anticipate and mitigate such impacts from substitutions or price changes.

Conclusions and future work

This study demonstrates that interval-valued fuzzy linear programming effectively balances nutritional adequacy and cost under uncertainty, achieving a minimum satisfaction level λ^*= 0.111 with strict pessimistic bounds. Energy adequacy emerged as the critical constraint, while crude-protein targets were more easily met, highlighting the importance of energy-dense ingredients. A weighted-average approach at w_{nut} = 0.25, further improved satisfaction λ= 0.272 at a moderate cost ₹ 28.7/kg, showing that slight prioritization of nutrition can enhance outcomes without drastic expense. Compared to deterministic methods, the interval-valued model achieved 65.7% higher minimal satisfaction while controlling costs. Sensitivity analyses confirmed predictable responses to fuzzy-interval adjustments and price fluctuations, enabling quantitative evaluation of ingredient substitutions.

By capturing data variability and expert hesitation through interval-valued membership functions, this work advances decision-support tools for animal nutrition. The model's transparent trade-off visualizations and scalable LP framework allow feed managers to make evidence-based adjustments efficiently. For practical adoption, we recommend using interval-valued goals, periodic parameter updates, and integrated dashboards for real-time optimization. Future research should explore dynamic multi-stage formulations, nonlinear membership functions, and type-2 fuzzy sets to address higher-order uncertainty, alongside system-level applications for mixed-species operations and stochastic price modeling.

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NOVELTY STATEMENT

This study introduces an innovative interval-valued fuzzy optimization framework for livestock feed formulation, addressing the critical challenge of balancing nutritional adequacy and economic cost under real-world variability. Unlike traditional deterministic or single-valued fuzzy approaches, our model captures both data uncertainty and expert hesitation through interval-valued membership functions, providing a more robust and adaptive decision-support tool. The framework explicitly quantifies trade-offs between nutrition and cost, enabling feed managers to calibrate solutions based on risk tolerance and market conditions. This approach significantly advances the field by enhancing solution robustness, transparency, and practicality in dynamic agricultural environments.

AUTHOR'S CONTRIBUTION

Suleiman Ibrahim Mohammad: Conceptualization, methodology, data curation, writing original draft. Dr. N Raja: Formal analysis, validation, writing review and editing. Asokan Vasudevan: Methodology, software implementation, visualization. Yogeesh N: Mathematical modeling, theoretical framework. Anber Abraheem Mohammad: Data collection, economic analysis. F. T. Z. Jabeen: Nutritional data validation, case study design. Mohammad Faleh Ahmmad Hunitie: Sensitivity analysis, benchmarking. Badrea Al Oraini: Interpretation, manuscript refinement, funding acquisition.

Conflict of interest

The authors have declared no conflict of interest.

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